1. There are 5 red balls, 4 blue balls, and 3 green balls in a bin. You draw two balls from a bin. What is the probability that

   (a) Ball 2 is red?

   (b) Ball 2 is red given that ball 1 is red, if the balls are drawn *with* replacement?

   (c) Ball 2 is red given that ball 1 is red, if the balls are drawn *without* replacement?

   (d) Ball 2 is not blue given that ball 1 is red, if the balls are drawn *without* replacement?

   (e) Ball 2 is red given that ball 1 is not blue, if the balls are drawn *without* replacement?

2. Alice, Bob, and Charlie are equally likely to have been born on any three days of the year. Let E_{AB} be the event that Alice and Bob were born on the same day. Define E_{BC} and E_{CA} analogously. Which of the following statements is true:

   (a) Any two of the three events E_{AB}, E_{BC}, E_{CA} are independent.

   (b) E_{AB}, E_{BC}, and E_{CA} are independent.

   (c) $E_{AB} \cup E_{BC}$ and E_{CA} are independent.
   ~~E_{AB} and E_{BC} are independent conditioned on E_{CA}.~~

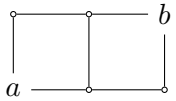
3. Cup 1 contains 3 blue balls and 2 red balls. Cup 2 contains 2 blue balls and 8 red balls. I choose a random cup and draw a ball from it.

   (a) What is the probability that it is blue?

   (b) The ball is blue. What is the probability that it came from cup 1?

   (c) I draw another ball from the same cup without replacement. What is the probability that it is also blue?

4. Computers a and b are linked through seven cables as in the picture. Each cable fails with probability 10% independently of the others. Let C be the event "there is a connection between a and b" and F be the event "the middle vertical cable fails".



   (a) What is the probability of C given F?

   (b) What is the probability of C given F^c?

   (c) What is the probability of C?

Additional ESTR 2018 questions

5. You have collected the following statistics on the fraction of smokers and drinkers that suffer from medical condition X among 1000 randomly tested individuals:

smokes	drinks	has X	number of cases
yes	yes	yes	3
yes	yes	no	27
yes	no	yes	7
yes	no	no	63
no	yes	yes	2
no	yes	no	68
no	no	yes	38

   Does drinking affect condition X?

6. Can there be four events E_1, E_2, E_3, E_4 so that every pair E_i, E_j is independent but every triple E_i, E_j, E_k is not (i, j, k are distinct indices)?

   More generally, suppose you are given a set $\mathcal{I}$ consisting of *subsets* of $\{1, \ldots, n\}$. Under which conditions on $\mathcal{I}$ can there exist a sample space Ω and events $E_1, \ldots, E_n$ such that for every set of indices I, the events $E_i \colon i \in I$ are independent when $I \in \mathcal{I}$ and not independent otherwise?