ENGG 2760A / ESTR 2018: Probability for Engineers — Homework 5

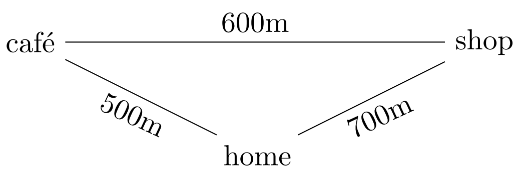
The Chinese University of Hong Kong, Fall 2022

1. You draw 4 balls without replacement from an urn with 3 blue balls and 3 red balls. Let $X$ be the number of blue balls in the draw.
   (a) Find the PMF and expected value of $X$.
   (b) Find the variance of $X$.
   (c) Let $X_i$ be an indicator for the event that the $i$-th ball is blue. What is $\mathrm{E}[X_i]$? Find $\mathrm{E}[X]$ again using linearity of expectation.

2. Roll a 4-sided die twice. Let $X$ be the larger value and $Y$ be the smaller value. Find (a) the joint PMF of $X$ and $Y$, (b) the marginal PMFs of $X$ and $Y$, (c) $\mathrm{E}[X + Y]$.

3. A six-sided die is rolled 6 times.
   (a) What is the probability that the value `1` occurs at least once?
   (b) Let $N$ be the number of distinct values that occur. For example, if the outcome is `521154`, then $N = 4$. What is $\mathrm{E}[N]$? (*Hint:* Write $N$ as a sum of indicators)

4. Alice can't find her expensive sweater. She estimates that there is a 30% chance that she left it at the café and a 40% chance that she left it at the shop (and that it is lost with the remaining probability). The distances between her home, the café, and the shop are given below. On her trip to find the sweater, in which order should she visit the venues so as to minimize her expected round-trip walking distance?




## Additional ESTR 2018 questions

5. Alice picks a sequence $(x_1, x_2, x_3)$ of 3 numbers from the set $S = \{-1, 0, 1\}$. Bob picks two positions $i < j$ in the sequence. Alice and Bob simultaneously reveal their choices to one another and $s$ dollars are transferred from Alice to Bob, where $s$ is the unique value in $S$ that is congruent to $x_j - x_i$ modulo 3. For example, if Alice picks $(1, -1, 1)$ and Bob picks $2 < 3$ then Alice pays Bob $x_3 - x_2 = 1 - (-1) \equiv -1$ dollars, that is she earns one dollar from Bob.

   Alice's strategy is to choose her sequence at random so that her <u>expected</u> earnings are maximizes, regardless of Bob's choice. How much probability should she assign to each sequence?

6. In Lecture 1 we also argued that a particle that moves at random one step left or right each time eventually revisits its origin. On average how many times will the particle visit the origin in the first 100 steps of its walk? Can you guess a rough formula for the expected number of returns to the origin in $n$ steps when $n$ is large?