The PDF of a delivery man's arrival hour past noon is

$$f(t) = \begin{cases} 1/(t+1)^2, & \text{if } t \geq 0 \\ 0, & \text{if } t < 0. \end{cases}$$

The delivery man hasn't arrived by 1pm. What is the probability that he arrives before 2pm?

Solution: Let T be the arrival time. The CDF of T is

$$\mathrm{P}(T \leq x) = \int_0^x \frac{1}{(t+1)^2} dt = \frac{1}{t+1}\Big|_0^x = 1 - \frac{1}{x+1}$$

for $x \geq 0$, so

$$\mathrm{P}(T < 2 | T > 1) = \frac{\mathrm{P}(1 < T < 2)}{\mathrm{P}(T > 1)} = \frac{\mathrm{P}(T < 2) - \mathrm{P}(T \leq 1)}{1 - \mathrm{P}(T \leq 1)} = \frac{1/2 - 1/3}{1/2} = \frac{1}{3}.$$