The arrival times of the next two buses are X and $X+Y$, respectively, where X and Y are independent Exponential random variables of rate 1. What is the covariance of X and $X+Y$? (An Exponential random variable of rate λ has expected value $1/\lambda$ and variance $1/\lambda^2$.)

Solution: We know $\mathrm{Cov}[X, X+Y] = \mathrm{E}[X(X+Y)] - \mathrm{E}[X]\,\mathrm{E}[X+Y]$ (4pts). Since X and Y are independent Exponential random variables of rate 1, $\mathrm{E}[X] = 1$ and $\mathrm{E}[X+Y] = \mathrm{E}[X] + \mathrm{E}[Y] = 2$ (2pts). $\mathrm{E}[X(X+Y)] = \mathrm{E}[X^2] + \mathrm{E}[XY] = \mathrm{Var}[X] + (\mathrm{E}[X])^2 + \mathrm{E}[X]E[Y] = 1 + 1 + 1 = 3$ (2pts). The final result is $\mathrm{Cov}[X, X+Y] = 3 - 2 = 1$ (2pts).